Joint resummation for slepton pair production at hadron colliders

Giuseppe Bozzi

Insitut für Theoretische Physik, Universität Karlsruhe, Postfach 6980, D-76128 Karlsruhe, Germany

Benjamin Fuks and Michael Klasen*

Laboratoire de Physique Subatomique et de Cosmologie,
Université Joseph Fourier/CNRS-IN2P3, 53 Avenue des Martyrs, F-38026 Grenoble, France
(Dated: February 5, 2008)

We present a precision calculation of the transverse-momentum and invariant-mass distributions for supersymmetric particle pair production at hadron colliders, focusing on Drell-Yan like slepton pair and slepton-sneutrino associated production at the CERN Large Hadron Collider. We implement the joint resummation formalism at the next-to-leading logarithmic accuracy with a process-independent Sudakov form factor, thus ensuring a universal description of soft-gluon emission, and consistently match the obtained result with the pure perturbative result at the first order in the strong coupling constant, i.e. at $\mathcal{O}(\alpha_s)$. We also implement three different recent parameterizations of non-perturbative effects. Numerically, we give predictions for $\tilde{e}_R \tilde{e}_R^*$ production and compare the resummed cross section with the perturbative result. The dependence on unphysical scales is found to be reduced, and non-perturbative contributions remain small.

PACS numbers: 12.60.Jv,13.85.Ni,14.80.Ly

I. INTRODUCTION

One of the main tasks in the experimental programme of the CERN Large Hadron Collider (LHC) is to perform an extensive and conclusive search of the supersymmetric (SUSY) partners of the Standard Model (SM) particles predicted by the Minimal Supersymmetric Standard Model [1, 2]. Scalar leptons are among the lightest supersymmetric particles in many SUSY-breaking scenarios [3, 4]. Presently, the experimental (lower) limits on electron, muon, and tau slepton masses are 73 GeV, 94 GeV, and 81.9 GeV, respectively [5]. Since sleptons often decay into the corresponding SM partner and the lightest stable SUSY particle, the distinctive signature at hadron colliders will consist in a highly energetic lepton pair and associated missing energy.

The leading order (LO) cross section for the production of non-mixing slepton pairs has been calculated in [6, 7, 8, 9], while the mixing between the interaction eigenstates was included in [10]. The next-to-leading order (NLO) QCD corrections have been calculated in [11], and the full SUSY-QCD corrections with non-mixing squarks in the loops have been added in [12]. Recently, an accurate calculation of the transverse-momentum (q_T) spectrum including soft-gluon resummation at the next-to-leading logarithmic (NLL) accuracy has been performed [13], allowing for the reconstruction of the mass and the determination of the spin of the produced particles by means of the Cambridge (s)transverse mass variable [14, 15] and for distinguishing thus the SUSY signal from the SM background, mainly due to WW and $t\bar{t}$ production [16, 17]. Very recently, the mixing effects relevant for the squarks appearing in the loops have been investigated at NLO, and the threshold-enhanced contributions have been computed at NLL [18]. The numerical results show a stabilization of the perturbative results through a considerable reduction of the scale dependence and a modest increase with respect to the NLO cross section.

Since the dynamical origin of the enhanced contributions is the same both in transverse-momentum and threshold resummations, i.e. the soft-gluon emission by the initial state, it would be desirable to have a formalism capable to handle at the same time the soft-gluon contributions in both the delicate kinematical regions, $q_T \ll M$ and $M^2 \sim s$, M being the slepton pair invariant-mass and s the partonic centre-of-mass energy. This *joint* resummation formalism has been developed in the last eight years [19, 20]. The exponentiation of the singular terms in the Mellin (N) and impact-parameter (b) space has been proven, and a consistent method to perform the inverse transforms, avoiding the Landau pole and the singularities of the parton distribution functions, has been introduced. Applications to prompt-photon [21], electroweak boson [22], Higgs boson [23], and heavy-quark pair [24] production at hadron colliders have exhibited substantial effects of joint resummation on the differential cross sections.

---

*klasen@lpsc.in2p3.fr

In this paper we apply the joint resummation formalism at the NLL level to the hadroproduction of slepton pairs at the LHC, thus completing our programme (started in Ref. [13] and continued in Ref. [18]) of providing the first precision calculations including soft-gluon resummation for slepton pair production at hadron colliders. In Sec. II, we briefly review the theoretical formalism of joint resummation following Refs. [20, 22]. We reorganize the terms of the resummed formula in a similar way as it was done for transverse-momentum resummation in [25]. The inverse transforms from the Mellin and impact-parameter spaces and the matching of the resummed result with the fixed-order perturbative results are discussed in Sec. III. Sec. IV is devoted to phenomenological predictions for the LHC, together with a comparison of the three types of resummation (transverse-momentum, threshold, and joint), showing their impact on the q_T-spectrum and on the invariant-mass distribution. Our results are summarized in Sec. V.

II. JOINT RESUMMATION AT THE NEXT-TO-LEADING LOGARITHMIC ORDER

We consider the hard scattering process

$$h_a(p_a) \, h_b(p_b) \rightarrow F(M, q_T) + X, \tag{1}$$

where F is a generic system of colourless particles, such as a Higgs boson or a Drell-Yan (s)lepton pair, M is the invariant mass of the final state F, and q_T is its transverse momentum. Thanks to the QCD factorization theorem, the unpolarized hadronic cross section

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M^2 \, \mathrm{d}q_T^2} = \sum_{a,b} \int_\tau^1 \mathrm{d}x_a \int_{\tau/x_a}^1 \mathrm{d}x_b \, f_{a/h_a}(x_a; \mu_F) \, f_{b/h_b}(x_b; \mu_F) \frac{\mathrm{d}^2\hat{\sigma}_{ab}}{\mathrm{d}M^2 \, \mathrm{d}q_T^2} (z; \alpha_s, \mu_R, \mu_F) \tag{2}$$

can be written as the convolution of the relevant partonic cross section $\hat{\sigma}_{ab}$ with the universal distribution functions $f_{a,b/h_{a,b}}$ of partons a, b inside the hadrons $h_{a,b}$, which depend on the longitudinal momentum fractions of the two partons $x_{a,b}$ and on the unphysical factorization scale μ_F. The partonic scattering cross section depends on the strong coupling constant α_s, the unphysical renormalization and factorization scales μ_R and μ_F, and on the scaling variable $z = M^2/s$, where $s = x_a x_b S$ and $S = (p_a + p_b)^2$ are the partonic and hadronic centre-of-mass energies, respectively. The lower limits for the integration over the longitudinal momentum fractions contain the quantity $\tau = M^2/S$, which approaches the value $\tau = 1$ when the process is close to the hadronic threshold $M^2 \sim S$. In Mellin N-space, the hadronic cross section naturally factorizes

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M^2 \, \mathrm{d}q_T^2} = \sum_{a,b} \oint_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i} \, \tau^{-N} \, f_{a/h_a}(N+1; \mu_F) \, f_{b/h_b}(N+1; \mu_F) \, \frac{\mathrm{d}^2\hat{\sigma}_{ab}}{\mathrm{d}M^2 \mathrm{d}q_T^2}(N; \alpha_s, \mu_R, \mu_F), \tag{3}$$

where the contour $\mathcal{C}$ in the complex N-space will be specified in Sec. III and the N-moments of the various quantities are defined according to the Mellin transform

$$F(N) = \int_0^1 \mathrm{d}x \, x^{N-1} \, F(x) \tag{4}$$

for $x = x_{a,b}, z, \tau$ and $F = f_{a/h_a, b/h_b}, \hat{\sigma}, \sigma$, respectively. The jointly resummed hadronic cross section in N-space can be written at NLL accuracy as [20, 22, 23]

$$\frac{\mathrm{d}^2\sigma^{\mathrm{(res)}}}{\mathrm{d}M^2 \mathrm{d}q_T^2}(N; \alpha_s, \mu_R, \mu_F) = \sum_c \hat{\sigma}_{c\bar{c}}^{(0)} \, H_{c\bar{c}}(\alpha_s, \mu_R) \int \frac{\mathrm{d}^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} \, \mathcal{C}_{c/h_a}(N, b; \alpha_s, \mu_R, \mu_F)$$

$$\times \, \exp\left[E_c^{\mathrm{(PT)}}(N, b; \alpha_s, \mu_R)\right] \, \mathcal{C}_{\bar{c}/h_b}(N, b; \alpha_s, \mu_F). \tag{5}$$

The indices c and $\bar{c}$ refer to the initial state of the lowest-order cross section $\hat{\sigma}_{c\bar{c}}^{(0)}$ and can then only be $q\bar{q}$ or gg, since the final state F is assumed to be colourless.

For slepton pair and slepton-sneutrino associated production at hadron colliders,

$$h_a(p_a) \, h_b(p_b) \rightarrow \tilde{l}_i(p_1)\tilde{l}_j^{(\prime)*}(p_2) + X, \tag{6}$$

we have $M^2 = (p_1 + p_2)^2$, $q_T^2 = (p_{1T} - p_{2T})^2$, and

$$\hat{\sigma}_{q\bar{q}}^{(0)} = \frac{\alpha^2 \pi \beta^3}{9\, M^2} \left[e_q^2 e_l^2 \,\delta_{ij} + \frac{e_q\, e_l\, \delta_{ij}(L_{qqZ} + R_{qqZ})\,\mathrm{Re}(L_{\tilde{l}_i\tilde{l}_j Z} + R_{\tilde{l}_i\tilde{l}_j Z})}{4\, x_W\,(1 - x_W)\,(1 - m_Z^2/M^2)} \right.$$

$$\left. + \frac{(L_{qqZ}^2 + R_{qqZ}^2)\,\left| L_{\tilde{l}_i\tilde{l}_j Z} + R_{\tilde{l}_i\tilde{l}_j Z} \right|^2}{32\, x_W^2\,(1 - x_W)^2(1 - m_Z^2/M^2)^2} \right], \tag{7}$$

$$\hat{\sigma}_{q\bar{q}'}^{(0)} = \frac{\alpha^2 \pi \beta^3}{9\, M^2} \left[\frac{\left| L_{qq'W} L_{\tilde{l}_i\tilde{\nu}_l W} \right|^2}{32\, x_W^2\,(1 - x_W)^2(1 - m_W^2/M^2)^2} \right], \tag{8}$$

where i, j denote slepton/sneutrino mass eigenstates with masses $m_{i,j}$, m_Z and m_W are the masses of the electroweak gauge bosons, α is the electromagnetic fine structure constant, $x_W = \sin^2\theta_W$ is the squared sine of the electroweak mixing angle, and the velocity β is defined as

$$\beta = \sqrt{1 + m_i^4/M^4 + m_j^4/M^4 - 2(m_i^2/M^2 + m_j^2/M^2 + m_i^2\, m_j^2/M^4)}. \tag{9}$$

The coupling strengths of the left- and right-handed (s)fermions to the electroweak vector bosons are given by

$$\{L_{ff'Z}, R_{ff'Z}\} = (2\, T_f^3 - 2\, e_f\, x_W) \times \delta_{ff'},$$

$$\{L_{\tilde{f}_i\tilde{f}_j Z}, R_{\tilde{f}_i\tilde{f}_j Z}\} = \{L_{ff'Z}\, S_{j1}^{\tilde{f}} S_{i1}^{\tilde{f}'*}, R_{ff'Z}\, S_{j2}^{\tilde{f}} S_{i2}^{\tilde{f}'*}\},$$

$$\{L_{qq'W}, R_{qq'W}\} = \{\sqrt{2}\, c_W\, V_{qq'}, 0\},$$

$$\{L_{\tilde{l}_i\tilde{\nu}_l W}, R_{\tilde{l}_i\tilde{\nu}_l W}\} = \{\sqrt{2}\, c_W\, S_{i1}^{\tilde{l}*}, 0\},$$

$$\{L_{\tilde{q}_i\tilde{q}_j' W}, R_{\tilde{q}_i\tilde{q}_j' W}\} = \{L_{qq'W} S_{i1}^{\tilde{q}*} S_{j1}^{\tilde{q}'}, 0\}, \tag{10}$$

where the weak isospin quantum numbers are $T_f^3 = \pm 1/2$ for left-handed and $T_f^3 = 0$ for right-handed (s)fermions, c_W is the cosine of the electroweak mixing angle, and $V_{ff'}$ are the CKM-matrix elements. The unitary matrices $S^{\tilde{f}}$ diagonalize the sfermion mass matrices, since in general the sfermion interaction eigenstates are not identical to the sfermion mass eigenstates (see App. A).

The function $H_{c\bar{c}}$ in Eq. (5) contains the hard virtual contributions and can be expanded perturbatively in powers of α_s,

$$H_{c\bar{c}}(\alpha_s, \mu_R) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu_R)}{\pi} \right)^n H_{c\bar{c}}^{(n)}(\mu_R). \tag{11}$$

The coefficients

$$\mathcal{C}_{c/h_a}(N, b; \alpha_s, \mu_R, \mu_F) = \sum_{a,b} C_{c/b}(N; \alpha_s(M/\chi))\, U_{b/a}(N; M/\chi, \mu_F)\, f_{a/h_a}(N+1; \mu_F) \tag{12}$$

and $\mathcal{C}_{\bar{c}/h_b}$, defined analogously, allow to evolve the parton distribution functions $f_{a,b/h_{a,b}}$ from the unphysical factorization scale μ_F to the physical scale M/χ with the help of the QCD evolution operator

$$U_{b/a}(N; \mu, \mu_0) = \exp\left[\int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}q^2}{q^2} \gamma_{b/a}(N; \alpha_s(q)) \right] \tag{13}$$

and to include, at this scale, the fixed-order contributions

$$C_{c/b}(N; \alpha_s) = \delta_{cb} + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^n C_{c/b}^{(n)}(N), \tag{14}$$

that become singular when $q_T \to 0$ (but not when $z \to 1$). The QCD evolution operator fulfils the differential equation

$$\frac{\mathrm{d}U_{b/a}(N; \mu, \mu_0)}{\mathrm{d}\ln\mu^2} = \sum_c U_{b/c}(N; \mu, \mu_0)\, \gamma_{c/a}(N; \alpha_s(\mu)), \tag{15}$$

where the anomalous dimensions $\gamma_{c/a}(N; \alpha_s)$ are the N-moments of the Altarelli-Parisi splitting functions. The function

$$\chi(\bar{b}, \bar{N}) = \bar{b} + \frac{\bar{N}}{1 + \eta\,\bar{b}/\bar{N}} \quad \text{with} \quad \bar{b} \equiv b\,M\,e^{\gamma_E}/2 \quad \text{and} \quad \bar{N} \equiv Ne^{\gamma_E} \tag{16}$$

organizes the logarithms of b and N in joint resummation. Its exact form is constrained by the requirement that the leading and next-to-leading logarithms in $\bar{b}$ and $\bar{N}$ are correctly reproduced in the limits $\bar{b} \to \infty$ and $\bar{N} \to \infty$, respectively. The choice of Eq. (16) with $\eta = 1/4$ avoids the introduction of sizeable subleading terms into perturbative expansions of the resummed cross section at a given order in α_s, which are not present in fixed-order calculations [22].

The perturbative (PT) eikonal exponent

$$E_c^{(\mathrm{PT})}(N, b; \alpha_s, \mu_R) = -\int_{M^2/\chi^2}^{M^2} \frac{\mathrm{d}\mu^2}{\mu^2} \left[A_c(\alpha_s(\mu)) \ln \frac{M^2}{\mu^2} + B_c(\alpha_s(\mu)) \right] \tag{17}$$

allows to resum soft radiation in the A-term, while the B-term accounts for the difference between the eikonal approximation and the full partonic cross section in the threshold region, i.e. the flavour-conserving collinear contributions. In the large-N limit, these coefficients are directly connected to the leading terms in the one-loop diagonal anomalous dimension calculated in the $\overline{\mathrm{MS}}$ factorization scheme [26]

$$\gamma_{c/c}(N; \alpha_s) = -A_c(\alpha_s) \ln \bar{N} - \frac{B_c(\alpha_s)}{2} + \mathcal{O}(1/N). \tag{18}$$

They can thus also be expressed as perturbative series in α_s,

$$A_c(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^n A_c^{(n)} \quad \text{and} \quad B_c(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^n B_c^{(n)}. \tag{19}$$

Performing the integration in Eq. (17), we obtain the form factor up to NLL,

$$E_c^{(\mathrm{PT})}(N, b; \alpha_s, \mu_R) = g_c^{(1)}(\lambda) \ln \chi + g_c^{(2)}(\lambda; \mu_R) \tag{20}$$

with

$$g_c^{(1)}(\lambda) = \frac{A_c^{(1)}}{\beta_0} \frac{2\lambda + \ln(1 - 2\lambda)}{\lambda},$$

$$g_c^{(2)}(\lambda; \mu_R) = \frac{A_c^{(1)} \beta_1}{\beta_0^3} \left[\frac{1}{2} \ln^2(1 - 2\lambda) + \frac{2\lambda + \ln(1 - 2\lambda)}{1 - 2\lambda} \right]$$

$$+ \left[\frac{A_c^{(1)}}{\beta_0} \ln \frac{M^2}{\mu_R^2} - \frac{A_c^{(2)}}{\beta_0^2} \right] \left[\frac{2\lambda}{1 - 2\lambda} + \ln(1 - 2\lambda) \right] + \frac{B_c^{(1)}}{\beta_0} \ln(1 - 2\lambda) \tag{21}$$

and $\lambda = \beta_0/\pi\,\alpha_s(\mu_R) \ln \chi$. The first two coefficients of the QCD β-function are

$$\beta_0 = \frac{1}{12}(11\,C_A - 4\,T_R\,N_f) \quad \text{and} \quad \beta_1 = \frac{1}{24}(17\,C_A^2 - 10\,T_R\,C_A\,N_f - 6\,C_F\,T_R\,N_f), \tag{22}$$

N_f being the number of effectively massless quark flavours and $C_F = 4/3$, $C_A = 3$, and $T_R = 1/2$ the usual QCD colour factors.

In order to explicitly factorize the dependence on the parameter χ, it is possible to reorganize the resummation of the logarithms in analogy to the case of transverse-momentum resummation [25, 27]. The hadronic resummed cross section can then be written as

$$\frac{\mathrm{d}^2\sigma^{(\mathrm{res})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2} = \sum_{a,b} \oint_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i} \tau^{-N} f_{a/h_a}(N+1; \mu_F)\, f_{b/h_b}(N+1; \mu_F) \int_0^{\infty} \frac{b\,\mathrm{d}b}{2} J_0(b\,q_T)$$

$$\times \sum_c \mathcal{H}_{ab \to c\bar{c}}\left(N; \alpha_s, \mu_R, \mu_F\right) \exp\left[\mathcal{G}_c(\ln \chi; \alpha_s, \mu_R)\right]. \tag{23}$$

The function $\mathcal{H}_{ab \to c\bar{c}}$ does not depend on the parameter χ and contains all the terms that are constant in the limits $b \to \infty$ or $N \to \infty$,

$$\mathcal{H}_{ab \to c\bar{c}}\Big(N; \alpha_s, \mu_R, \mu_F\Big) = \hat{\sigma}_{c\bar{c}}^{(0)} \left[\delta_{ca}\delta_{\bar{c}b} + \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu_R)}{\pi} \right)^n \mathcal{H}_{ab \to c\bar{c}}^{(n)}\Big(N; \mu_R, \mu_F\Big) \right]. \tag{24}$$

At $\mathcal{O}(\alpha_s)$, the coefficient $\mathcal{H}_{ab \to c\bar{c}}^{(1)}$ is given by

$$\mathcal{H}_{ab \to c\bar{c}}^{(1)}\Big(N; \mu_R, \mu_F\Big) = \delta_{ca}\delta_{\bar{c}b}\, H_{c\bar{c}}^{(1)}(\mu_R) + \delta_{ca}\, C_{\bar{c}/b}^{(1)}(N) + \delta_{\bar{c}b}\, C_{c/a}^{(1)}(N) + \Big(\delta_{ca}\gamma_{\bar{c}/b}^{(1)}(N) + \delta_{\bar{c}b}\gamma_{c/a}^{(1)}(N) \Big) \ln \frac{M^2}{\mu_F^2}. \tag{25}$$

The χ-dependence appearing in the C-coefficient and in the evolution operator U of Eq. (12) is factorized into the exponent $\mathcal{G}_c$, which has the same form as $E_c^{(\mathrm{PT})}$ defined in Eq. (17) except for the substitution

$$B_c(\alpha_s) \to \tilde{B}_c(N; \alpha_s) = B_c(\alpha_s) + 2\beta(\alpha_s) \frac{\mathrm{d} \ln C_{c/c}(N; \alpha_s)}{\mathrm{d} \ln \alpha_s} + 2\,\gamma_{c/c}(N; \alpha_s). \tag{26}$$

At NLL accuracy, Eq. (20) remains almost unchanged, since only the coefficient $g_c^{(2)}$ of Eq. (21) has to be slightly modified by

$$B_c^{(1)} \to \tilde{B}_c^{(1)}(N) = B_c^{(1)} + 2\gamma_{c/c}^{(1)}(N). \tag{27}$$

Although the first-order coefficients $C_{a/b}^{(1)}(N)$ and $H_{c\bar{c}}^{(1)}(\mu_R)$ are in principle resummation-scheme dependent [27], this dependence cancels in the perturbative expression of $\mathcal{H}_{ab \to c\bar{c}}$ [25]. In the numerical code we developed for slepton pair production, we implement the Drell-Yan resummation scheme and take $H_{q\bar{q}}(\alpha_s, \mu_R) \equiv 1$. The C-coefficients are then given by

$$C_{q/q}^{(1)}(N) = \frac{2}{3\,N\,(N+1)} + \frac{\pi^2 - 8}{3} \quad \text{and} \quad C_{q/g}^{(1)}(N) = \frac{1}{2\,(N+1)\,(N+2)}. \tag{28}$$

III. INVERSE TRANSFORM AND MATCHING WITH THE PERTURBATIVE RESULT

Once resummation has been achieved in N- and b-space, inverse transforms have to be performed in order to get back to the physical spaces. Special attention has to be paid to the singularities in the resummed exponent, related to the divergent behaviour near $\chi = \exp[\pi/(2\beta_0\alpha_s)]$, i.e. the Landau pole of the running strong coupling, and near $\bar{b} = -2\bar{N}$ and $\bar{b} = -4\bar{N}$, where $\chi = 0$ and infinity, respectively. The integration contours of the inverse transforms in the Mellin and impact parameter spaces must therefore avoid hitting any of these poles.

The $b-$integration is performed by deforming the integration contour with a diversion into the complex b-space [21], defining two integration branches

$$b = (\cos\varphi \pm i \sin\varphi)t \quad \text{with} \ 0 \le t \le \infty, \tag{29}$$

valid under the condition that the integrand decreases sufficiently rapidly for large values of $|b|$. The Bessel function J_0 is replaced by two auxiliary functions $h_{1,2}(z, v)$ related to the Hankel functions

$$h_1(z, v) \equiv -\frac{1}{\pi} \int_{-iv\pi}^{-\pi+iv\pi} \mathrm{d}\theta\, e^{-iz\sin\theta},$$

$$h_2(z, v) \equiv -\frac{1}{\pi} \int_{\pi+iv\pi}^{-iv\pi} \mathrm{d}\theta\, e^{-iz\sin\theta}. \tag{30}$$

Their sum is always $h_1(z, v) + h_2(z, v) = 2\,J_0(z)$, but they distinguish positive and negative phases of the b-contour, being then associated with only one of the two branches defined in Eq. (29).

The inverse Mellin transform is performed following a contour inspired by the Minimal Prescription [28] and the Principal Value Resummation [29], where one again defines two branches

$$N = C + z\,e^{\pm i\phi} \quad \text{with} \ 0 \le z \le \infty, \ \pi > \phi > \frac{\pi}{2}. \tag{31}$$

The parameter C is chosen in such a way that all the singularities related to the N-moments of the parton densities are to the left of the integration contour. It has to lie within the range $0 < C < \exp[\pi/(2\beta_0\alpha_s) - \gamma_E]$ in order to obtain convergent inverse transform integrals for any choice of ϕ and φ.

A matching procedure of the NLL resummed cross section to the NLO result has to be performed in order to keep the full information contained in the fixed-order calculation and to avoid possible double-counting of the logarithmically enhanced contributions. A correct matching is achieved through the formula

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M^2\,\mathrm{d}q_T^2} = \frac{\mathrm{d}^2\sigma^{\mathrm{(F.O.)}}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(\alpha_s) + \oint_{C_N} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N} \int \frac{b\,\mathrm{d}b}{2} J_0(b\,q_T) \left[\frac{\mathrm{d}^2\sigma^{\mathrm{(res)}}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N,b;\alpha_s) - \frac{\mathrm{d}^2\sigma^{\mathrm{(exp)}}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N,b;\alpha_s) \right], \tag{32}$$

where $\mathrm{d}^2\sigma^{\mathrm{(F.O.)}}$ is the fixed-order perturbative result, $\mathrm{d}^2\sigma^{\mathrm{(res)}}$ is the resummed cross section discussed above, and $\mathrm{d}^2\sigma^{\mathrm{(exp)}}$ is the truncation of the resummed cross section to the same perturbative order as $\mathrm{d}^2\sigma^{\mathrm{(F.O.)}}$. Here, we have removed the scale dependences for brevity.

At NLO, the double-differential partonic cross section

$$\frac{\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{(F.O.)}}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(z;\alpha_s,\mu_R) = \delta(q_T^2)\,\delta(1-z)\,\hat{\sigma}_{ab}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\,\hat{\sigma}_{ab}^{(1)}(z) + \mathcal{O}(\alpha_s^2) \tag{33}$$

receives contributions from the emission of an extra gluon jet and from processes with an initial gluon splitting into a $q\bar{q}$ pair,

$$\hat{\sigma}_{qg}^{(1)}(z) = \frac{T_R}{2\,s} A_{qg}(s,t,u)\, \sigma_{q\bar{q}^{(\prime)}}^{(0)}, \tag{34}$$

$$\hat{\sigma}_{g\bar{q}}^{(1)}(z) = \frac{T_R}{2\,s} A_{qg}(s,u,t)\, \sigma_{q\bar{q}^{(\prime)}}^{(0)}, \tag{35}$$

$$\hat{\sigma}_{q\bar{q}^{(\prime)}}^{(1)}(z) = \frac{C_F}{2\,s} A_{qq}(s,t,u)\, \sigma_{q\bar{q}^{(\prime)}}^{(0)}(M) \tag{36}$$

with [30]

$$A_{qg}(s,t,u) = -\left(\frac{s}{t} + \frac{t}{s} + \frac{2u\,M^2}{st} \right), \tag{37}$$

$$A_{qq}(s,t,u) = -A_{qg}(u,t,s). \tag{38}$$

The Mandelstam variables s, t, and u refer to the $2 \to 2$ scattering process $ab \to \gamma,\ Z^0,\ W^\pm + X$ and are related to the invariant mass M (or scaled squared invariant mass $z = M^2/s$), transverse momentum q_T, and rapidity y of the slepton pair by the well-known relations

$$s = x_a x_b S = M^2/z, \tag{39}$$

$$t = M^2 - \sqrt{S(M^2+q_T^2)}x_b e^y, \tag{40}$$

$$u = M^2 - \sqrt{S(M^2+q_T^2)}x_a e^{-y}. \tag{41}$$

Integration over q_T requires the cancellation of soft and collinear singularities with virtual contributions in order to arrive at the finite single-differential partonic cross section

$$\frac{\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{(F.O.)}}}{\mathrm{d}M^2}(z;\alpha_s,\mu_R,\mu_F) = \hat{\sigma}_{ab}^{(0)}\,\delta(1-z) + \frac{\alpha_s(\mu_R)}{\pi}\,\hat{\sigma}_{ab}^{(1)}(z;\mu_R,\mu_F) + \mathcal{O}(\alpha_s^2), \tag{42}$$

where the first term $\hat{\sigma}_{ab}^{(0)}$ is defined in Eqs. (7) and (8) and the second term including the full NLO SUSY-QCD corrections can be found in Ref. [18].

The expansion of the resummed result reads

$$\frac{\mathrm{d}^2\sigma^{\mathrm{(exp)}}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N,b;\alpha_s,\mu_R,\mu_F) = \sum_{a,b} f_{a/h_a}(N+1;\mu_F)\, f_{b/h_b}(N+1;\mu_F)\, \hat{\sigma}_{ab}^{\mathrm{(exp)}}(N,b;\alpha_s,\mu_R,\mu_F), \tag{43}$$

where $\hat{\sigma}_{ab}^{\mathrm{(exp)}}$ is obtained by perturbatively expanding the resummed component

$$\hat{\sigma}_{ab}^{\mathrm{(exp)}}(N,b;\alpha_s,\mu_R,\mu_F) = \sum_c \hat{\sigma}_{c\bar{c}}^{(0)} \left\{ \delta_{ca}\delta_{\bar{c}b} + \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu_R)}{\pi} \right)^n \left[\tilde{\Sigma}_{ab\to c\bar{c}}^{(n)}(N,\ln\chi;\mu_R,\mu_F) \right. \right.$$

$$\left. \left. + \mathcal{H}_{ab\to c\bar{c}}^{(n)}\left(N;\mu_R,\mu_F\right) \right] \right\}. \tag{44}$$

The perturbative coefficients $\tilde{\Sigma}^{(n)}$ are polynomials of degree $2n$ in $\ln\chi$, and $\mathcal{H}^{(n)}$ embodies the constant part of the resummed cross section in the limits $b \to \infty$ and $N \to \infty$. In particular, the first-order coefficient $\tilde{\Sigma}^{(1)}$ is given by

$$\tilde{\Sigma}^{(1)}_{ab \to c\bar{c}}(N, \ln\chi) = \tilde{\Sigma}^{(1;2)}_{ab \to c\bar{c}} \ln^2\chi + \tilde{\Sigma}^{(1;1)}_{ab \to c\bar{c}}(N) \ln\chi, \tag{45}$$

with

$$\tilde{\Sigma}^{(1;2)}_{ab \to c\bar{c}} = -2\, A_c^{(1)} \delta_{ca}\delta_{\bar{c}b} \quad \text{and} \quad \tilde{\Sigma}^{(1;1)}_{ab \to c\bar{c}}(N) = -2\left(B_c^{(1)} \delta_{ca}\delta_{\bar{c}b} + \delta_{ca}\gamma^{(1)}_{\bar{c}/b}(N) + \delta_{\bar{c}b}\gamma^{(1)}_{c/a}(N)\right). \tag{46}$$

IV. NUMERICAL RESULTS

We now present numerical results for the production of a right-handed selectron pair at the LHC for a centre-of-mass energy of $\sqrt{S} = 14$ TeV. For the masses and widths of the electroweak gauge bosons and the mass of the top quark, we use the values $m_Z = 91.1876$ GeV, $m_W = 80.403$ GeV, $\Gamma_Z = 2.4952$ GeV, $\Gamma_W = 2.141$ GeV, and $m_t = 174.2$ GeV [5]. The electromagnetic fine structure constant $\alpha = \sqrt{2}G_F m_W^2 \sin^2\theta_W/\pi$ is calculated in the improved Born approximation using the world average value $G_F = 1.16637 \cdot 10^{-5}$ GeV^{-2} for Fermi's coupling constant, and $\sin^2\theta_W = 1 - m_W^2/m_Z^2$.

We choose the mSUGRA benchmark point BFHK B [31], which gives after the renormalization group evolution of the SUSY-breaking parameters [32] a light $\tilde{e}_R$ of mass $m_{\tilde{e}_R} = 186$ GeV and rather heavy squarks with masses around 800-850 GeV. The top-squark mass eigenstate $\tilde{t}_1$ is slightly lighter, but does not contribute to the virtual squark loops due to the negligible top-quark density in the proton. For the LO (NLO and NLL) predictions, we use the LO 2001 [33] (NLO 2004 [34]) MRST sets of parton distribution functions. For the NLO and NLL predictions, α_s is evaluated with the corresponding value of $\Lambda^{n_f=5}_{\overline{\text{MS}}} = 255$ MeV at two-loop accuracy. We allow the unphysical scales μ_F and μ_R to vary between $M/2$ and $2M$ to estimate the perturbative uncertainty.

In Fig. 1, we present the transverse-momentum spectrum of the selectron pair, obtained after integrating the equations of Sec. II and Sec. III over M^2, from the $\tilde{e}_R\tilde{e}_R^*$ production threshold up to the hadronic centre-of-mass energy. We plot the fixed order result at order α_s (dashed line), the expansion of the resummed formula at the same perturbative order (dotted line), the total NLL+LO matched result (solid line), and the uncertainty bands from the scale variation. The fixed order result diverges as expected as q_T tends to zero. The asymptotic expansion of the resummation formula is in good agreement with the $\mathcal{O}(\alpha_s)$ result in this kinematical region, since the cross section is dominated by the large logarithms that we are resumming. For intermediate values of q_T, we can see that the agreement between the expansion and the perturbative result is slightly worse. This effect was not present in q_T-resummation for such q_T-values [13] and is thus related to the threshold-enhanced contributions important in the large-M region. This can also be seen in Fig. 2, where we directly compare the jointly- and q_T-matched results, the latter having been obtained with the q_T-resummation formalism of Ref. [25]. The two approaches lead to a similar behaviour in the small-q_T region, but the jointly-resummed cross section is about 5%-10% lower than the q_T-resummed cross section for transverse momenta in the range 50 GeV $< q_T < 100$ GeV. However, the effect of the resummation is clearly visible in both cases, the resummation-improved result being even 40% higher than the fixed-order result at $q_T = 80$ GeV. In Fig. 1, we also estimate the theoretical uncertainties through an independent variation of the factorization and renormalization scales between $M/2$ and $2M$ and show that the use of resummation leads to a clear improvement with respect to the fixed-order calculation. In the small and intermediate q_T-regions the scale variation amounts to 10% for the fixed-order result, while it is always less than 5% for the matched result.

The q_T-distribution is affected by non-perturbative effects in the small q_T-region coming, for instance, from partons with a non-zero intrinsic transverse-momentum inside the hadron and from unresolved gluons with very small transverse momentum. Global fits of experimental Drell-Yan data allow for different parameterizations of these effects, which can be consistently included in the resummation formula of Eq. (23) through a non-perturbative form factor F_{ab}^{NP}. We include in our analysis three different parameterizations of this factor [35, 36, 37],

$$F_{ab}^{\text{NP}(LY-G)}(b, M, x_1, x_2) = \exp\left[-b^2\left(\bar{g}_1 + \bar{g}_2 \ln\frac{b_{\max}M}{2}\right) - b\,\bar{g}_1\,\bar{g}_3\,\ln(100\,x_1 x_2)\right], \tag{47}$$

$$F_{ab}^{\text{NP}(BLNY)}(b, M, x_1, x_2) = \exp\left[-b^2\left(\tilde{g}_1 + \tilde{g}_2 \ln\frac{b_{\max}M}{2} + \tilde{g}_1\,\tilde{g}_3\,\ln(100\,x_1 x_2)\right)\right], \tag{48}$$

$$F_{ab}^{\text{NP}(KN)}(b, M, x_1, x_2) = \exp\left[-b^2\left(a_1 + a_2 \ln\frac{M}{3.2\text{GeV}} + a_3 \ln(100\,x_1 x_2)\right)\right]. \tag{49}$$

The most recent values for the free parameters in these functions can be found in Refs. [36, 37]. We show in the

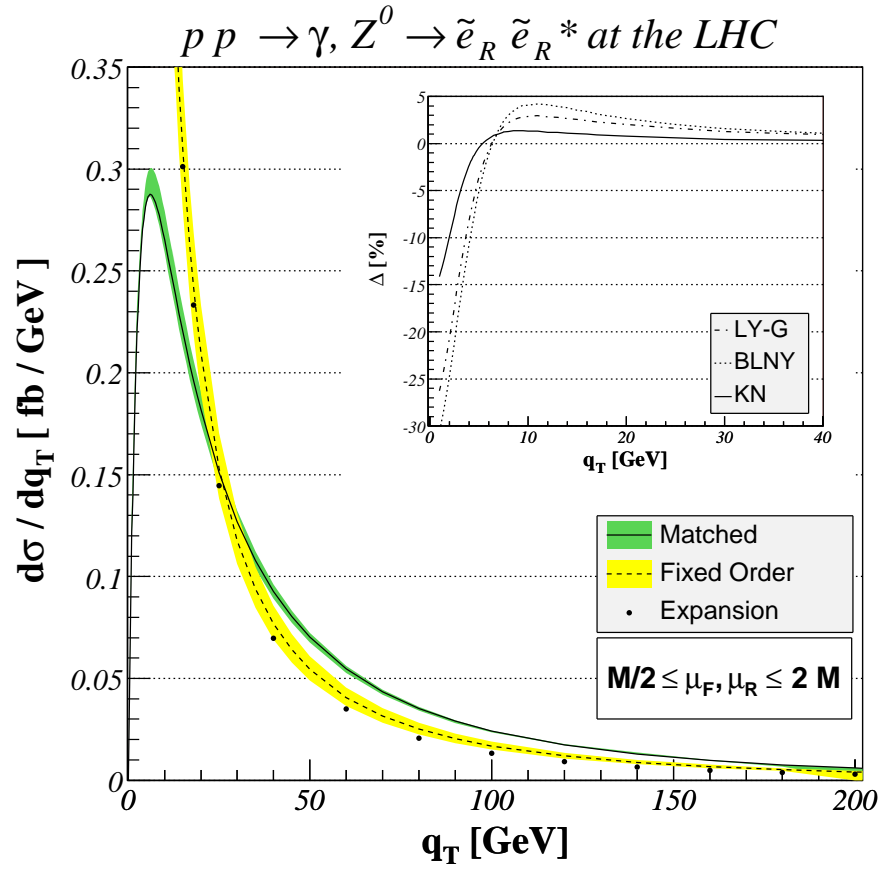## $p\,p \rightarrow \gamma, Z^0 \rightarrow \tilde{e}_R\,\tilde{e}_R{}^*$ at the LHC

FIG. 1: Transverse-momentum distribution for the process $pp \rightarrow \tilde{e}_R\,\tilde{e}_R^*$ at the LHC. NLL+LO matched (full), fixed order (dotted) and asymptotically expanded results are shown, together with three different parameterizations of non-perturbative effects (insert).
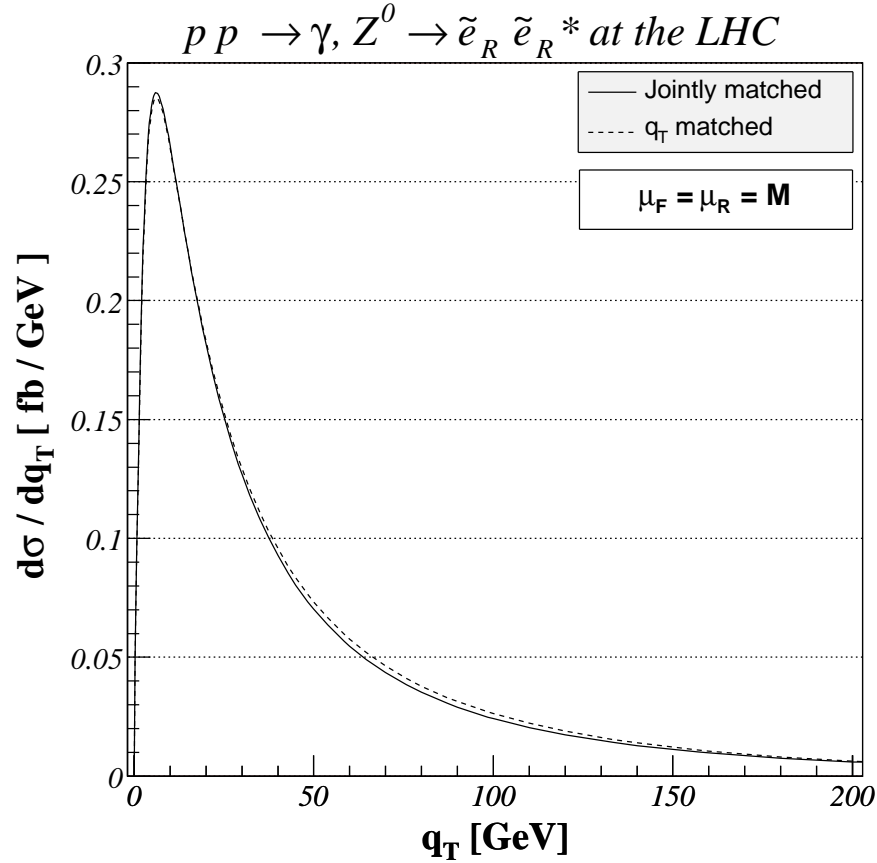
$p\,p \rightarrow \gamma, Z^0 \rightarrow \tilde{e}_R\,\tilde{e}_R{}^*$ at the LHC

FIG. 2: Transverse-momentum distribution of selectron pairs at the LHC in the framework of joint (full) and q_T (dotted)

upper-right part of Fig. 1 the quantity

$$\Delta = \frac{d\sigma^{(\text{res.+NP})}(\mu_R = \mu_F = M) - d\sigma^{(\text{res.})}(\mu_R = \mu_F = M)}{d\sigma^{(\text{res.})}(\mu_R = \mu_F = M)},$$ (50)

which gives thus an estimate of the contributions from the different NP parameterizations (LY-G, BLNY and KN). They are under good control, since they are always less than 5% for $q_T > 5$ GeV and thus considerably smaller than the resummation effects.

The invariant-mass distribution $M^3 d\sigma/dM$ for $\tilde{e}_R$-pair production at the LHC is obtained after integrating the equations of Sec. II and Sec. III over q_T^2 and is shown in Fig. 3. The differential cross section $d\sigma/dM$ has been multiplied by a factor M^3 in order to remove the leading mass dependence of propagator and phase space factors. We can see the P-wave behaviour relative to the pair production of scalar particles, since the invariant-mass distribution rises above the threshold at $\sqrt{s} = 2m_{\tilde{e}_R}$ with the third power of the slepton velocity and peaks at about 200 GeV above threshold (both for $M^3 d\sigma/dM$ and the not shown $d\sigma/dM$ differential distribution), before falling off steeply due to the s-channel propagator and the decreasing parton luminosity. In the large-M region, the resummed cross section is 30% higher than the leading order cross section, but this represents only a 3% increase with respect to the NLO SUSY-QCD result. In the small-M region, much further then from the hadronic threshold, resummation effects are rather limited, inducing a modification of the NLO results smaller than 1%. The shaded, horizontally, and vertically hashed bands in Fig. 3 represent the theoretical uncertainties for the LO, NLO SUSY-QCD, and the jointly-matched predictions. At LO the dependence comes only from the factorization scale and increases with the momentum-fraction x of the partons in the proton (i.e. with M), being thus larger in the right part of the figure. This dependence is largely reduced at NLO due to the factorization of initial-state singularities in the PDFs. Including the dependence due to the renormalization scale in the coupling $\alpha_s(\mu_R)$, the total variation is about 7%-11%. After resummation, the total scale uncertainty is finally reduced to only 7%-8% for the matched result, the reduction being of course more important in the large-M region, where the resummation effects are more important.

In Fig. 4, we show the cross section correction factors

$$K^i = \frac{d\sigma^i/dM}{d\sigma^{\text{LO}}/dM}$$ (51)

as a function of the invariant-mass M. i labels the corrections induced by NLO QCD, NLO SUSY-QCD, joint- and threshold-resummation (as obtained in [18]), these two last calculations being matched with the NLO SUSY-QCD result. At small invariant mass M, the resummation is less important, since we are quite far from the hadronic threshold, as shown in the left part of the plot. At larger M, the logarithms become important and lead to a larger increase of the resummed K-factors over the fixed-order one. We also show the difference between threshold and joint resummations, which is only about one or two percents. This small difference is due to the choice of the Sudakov form factor $\mathcal{G}$ and of the $\mathcal{H}$-function, which correctly reproduce transverse-momentum resummation in the limit of $b \to \infty$, N being fixed, but which present some differences in the pure threshold limit $b \to 0$ and $N \to \infty$, as it was the case for joint resummation for Higgs and electroweak boson production [22, 23]. However, this effect is under good control, since it is much smaller than the theoretical scale uncertainty of about 7%.

V. CONCLUSIONS

With this work we complete our programme of performing precision calculations for slepton pair production at hadron colliders. Together with the previous papers on transverse-momentum [13] and threshold [18] resummation, soft-gluon resummation effects are now consistently included in predictions for various distributions exploiting the q_T, threshold, and joint resummation formalisms. We found that the effects obtained from resumming the enhanced soft contributions are important at hadron colliders, even far from the critical kinematical regions where the resummation procedure is fully justified. The numerical results show a considerable reduction of the scale uncertainty with respect to fixed order results and also a negligible dependence on non-perturbative effects, introduced through different Gaussian-like smearings of the Sudakov form factors. These features lead to an increased stability of the perturbative results and thus to a possible improvement of the slepton pair (slepton-sneutrino) search strategies at the LHC.
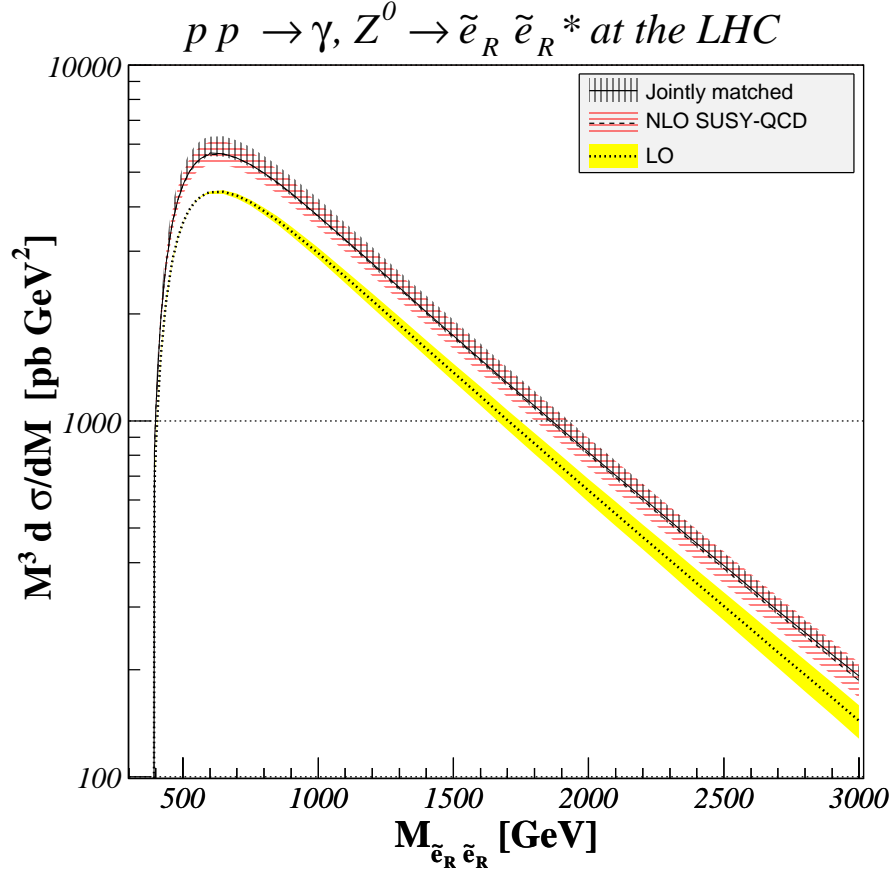
FIG. 3: Invariant-mass distribution $M^3 \, \mathrm{d}\sigma/\mathrm{d}M$ of $\tilde{e}_R$ pairs at the LHC. We show the total NLL+NLO jointly matched (full), as well as the fixed-order NLO SUSY-QCD (dashed) and LO QCD (dotted) results, with the corresponding scale uncertainties (vertically hashed, horizontally hashed, and shaded bands).
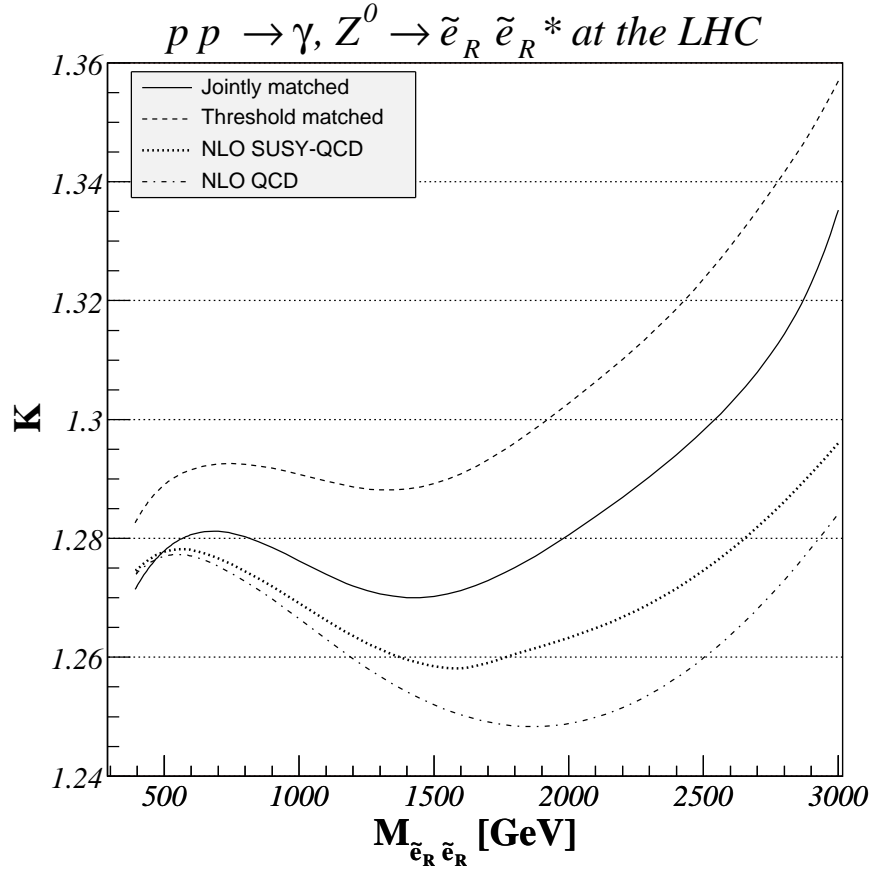

FIG. 4: K-factors as defined in Eq. (51) for $\tilde{e}_R$ pair production at the LHC. We show the total NLL+NLO jointly (full), and

APPENDIX A: SFERMION MIXING

The soft SUSY-breaking terms A_f of the trilinear Higgs-sfermion-sfermion interaction and the off-diagonal Higgs mass parameter μ in the MSSM Lagrangian induce mixings of the left- and right-handed sfermion eigenstates $\tilde{f}_{L,R}$ of the electroweak interaction into mass eigenstates $\tilde{f}_{1,2}$. The sfermion mass matrix is given by [2]

$$\mathcal{M}^2 \;=\; \begin{pmatrix} m_{LL}^2 + m_f^2 & m_f\,m_{LR}^* \\ m_f\,m_{LR} & m_{RR}^2 + m_f^2 \end{pmatrix} \tag{A1}$$

with

$$m_{LL}^2 \;=\; m_{\tilde{F}}^2 + (T_f^3 - e_f\,\sin^2\theta_W)\,m_Z^2\,\cos 2\beta, \tag{A2}$$

$$m_{RR}^2 \;=\; m_{\tilde{F}'}^2 + e_f\,\sin^2\theta_W\,m_Z^2\,\cos 2\beta, \tag{A3}$$

$$m_{LR} \;=\; A_f - \mu^* \begin{cases} \cot\beta & \text{for up} - \text{type sfermions.} \\ \tan\beta & \text{for down} - \text{type sfermions.} \end{cases} \tag{A4}$$

It is diagonalized by a unitary matrix $S^{\tilde{f}}$, $S^{\tilde{f}}\,\mathcal{M}^2\,S^{\tilde{f}\dagger} = \text{diag}\,(m_1^2, m_2^2)$, and has the squared mass eigenvalues

$$m_{1,2}^2 = m_f^2 + \frac{1}{2}\Big(m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4\,m_f^2\,|m_{LR}|^2}\Big). \tag{A5}$$

For real values of m_{LR}, the sfermion mixing angle $\theta_{\tilde{f}}$, $0 \leq \theta_{\tilde{f}} \leq \pi/2$, in

$$S^{\tilde{f}} = \begin{pmatrix} \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} \\ -\sin\theta_{\tilde{f}} & \cos\theta_{\tilde{f}} \end{pmatrix} \text{ with } \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = S^{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} \tag{A6}$$

can be obtained from

$$\tan 2\theta_{\tilde{f}} = \frac{2\,m_f\,m_{LR}}{m_{LL}^2 - m_{RR}^2}. \tag{A7}$$

If m_{LR} is complex, one may first choose a suitable phase rotation $\tilde{f}_R' = e^{i\phi}\tilde{f}_R$ to make the mass matrix real and then diagonalize it for $\tilde{f}_L$ and $\tilde{f}_R'$. $\tan\beta = v_u/v_d$ is the (real) ratio of the vacuum expectation values of the two Higgs fields, which couple to the up-type and down-type (s)fermions. The soft SUSY-breaking mass terms for left- and right-handed sfermions are $m_{\tilde{F}}$ and $m_{\tilde{F}'}$ respectively.

[1] H. P. Nilles, Phys. Rept. **110** (1984) 1.
[2] H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75.
[3] B. C. Allanach *et al.*, Eur. Phys. J. C **25** (2002) 113.
[4] J. A. Aguilar-Saavedra *et al.*, Eur. Phys. J. C **46** (2006) 43.
[5] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33** (2006) 1.
[6] S. Dawson, E. Eichten and C. Quigg, Phys. Rev. D **31** (1985) 1581.
[7] P. Chiappetta, J. Soffer and P. Taxil, Phys. Lett. B **162** (1985) 192.
[8] F. del Aguila and L. Ametller, Phys. Lett. B **261** (1991) 326.
[9] H. Baer, C. H. Chen, F. Paige and X. Tata, Phys. Rev. D **49** (1994) 3283.
[10] G. Bozzi, B. Fuks and M. Klasen, Phys. Lett. B **609** (2005) 339.
[11] H. Baer, B.W. Harris and M.H. Reno, Phys. Rev. D **57** (1998) 5871.
[12] W. Beenakker, M. Klasen, M. Krämer, T. Plehn, M. Spira and P. M. Zerwas, Phys. Rev. Lett. **83** (1999) 3780.
[13] G. Bozzi, B. Fuks and M. Klasen, Phys. Rev. D **74** (2006) 015001.
[14] C. G. Lester and D. J. Summers, Phys. Lett. B **463** (1999) 99.
[15] A. J. Barr, JHEP **0602** (2006) 042.
[16] E. Lytken, Czech. J. Phys. **54** (2004) A169.
[17] Yu. M. Andreev, S. I. Bityukov and N. V. Krasnikov, Phys. Atom. Nucl. **68** (2005) 340.
[18] G. Bozzi, B. Fuks and M. Klasen, Nucl. Phys. B **777** (2007) 157.
[19] H. N. Li, Phys. Lett. B **454** (1999) 328.
[20] E. Laenen, G. Sterman and W. Vogelsang, Phys. Rev. D **63** (2001) 114018.
[21] E. Laenen, G. Sterman and W. Vogelsang, Phys. Rev. Lett. **84** (2000) 4296.

[22] A. Kulesza, G. Sterman and W. Vogelsang, Phys. Rev. D **66** (2002) 014011.

[23] A. Kulesza, G. Sterman and W. Vogelsang, Phys. Rev. D **69** (2004) 014012.

[24] A. Banfi and E. Laenen, Phys. Rev. D **71** (2005) 034003.

[25] G. Bozzi, S. Catani, D. de Florian and M. Grazzini, Nucl. Phys. B **737** (2006) 73.

[26] G. P. Korchemsky, Mod. Phys. Lett. A **4** (1989) 1257.

[27] S. Catani, D. de Florian and M. Grazzini, Nucl. Phys. B **596** (2001) 299.

[28] S. Catani, M. L. Mangano, P. Nason and L. Trentadue, Nucl. Phys. B **478** (1996) 273.

[29] H. Contopanagos and G. Sterman, Nucl. Phys. B **419** (1994) 77.

[30] R. J. Gonsalves, J. Pawlowski and C. F. Wai, Phys. Rev. D **40** (1989) 2245.

[31] G. Bozzi, B. Fuks, B. Herrmann and M. Klasen, arXiv:0704.1826 [hep-ph].

[32] W. Porod, Comput. Phys. Commun. **153** (2003) 275.

[33] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Phys. Lett. B **531** (2002) 216.

[34] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Phys. Lett. B **604** (2004) 61.

[35] G. A. Ladinsky and C. P. Yuan, Phys. Rev. D **50** (1994) 4239.

[36] F. Landry, R. Brock, P. M. Nadolsky and C. P. Yuan, Phys. Rev. D **67** (2003) 073016.

[37] A. V. Konychev and P. M. Nadolsky, Phys. Lett. B **633** (2006) 710.